POD Study of a Turbulent Boundary Layer over a Rough Forward-facing Step

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Introduction

Turbulent flows over forward-facing steps (FFSs) have been the focus of many studies due to their importance in understanding the generation of noise and structural vibrations in various engineering applications. The unique features of the turbulent flows over a forward-facing step include the existence of two recirculation regions: one is located in the lower front corner of the step and the other one occurs downstream of the step's front and on the top surface of the step. A strong shear layer which is representative of a mixing layer exists above the downstream recirculation region.

Near-wall turbulence statistics of the boundary layers over the FFS are investigated in few studies of those include Hattori and Nagano (2010) using DNS (Re_h = 900 ~ 3000, $\delta/h \approx$ 1.5 ~ 3) and in Ren and Wu (2011) through PIV measurements (Re_h = 3450, δ/h = 8). In both studies, a significant increase of Reynolds normal and shear stresses was observed in the downstream recirculation region. In addition, their quadrant analysis revealed that the the ejection events still contributed more to the average Reynolds shear stress than the sweep events although most ejections occurred further away from the top wall of the step while sweeps occurred most often closer to the step's surface. Ren and Wu (2011) also investigated the effects of realistic roughness topography on the turbulent boundary layer over the forward-facing step. A rough FFS flow can be used to model an elevated rough surface at the surface discontinuity which may be encountered in engineering flows such as those over a rough turbine blade in aircraft engines or in power plants. Their results showed that the separated turbulent flow in the downstream recirculation region had been significantly modified by the studied roughness.

Even far less explored are the turbulent structures in the turbulent FFS flows. The objective of this work is to investigate the impacts of a realistic roughness on the coherent turbulence structures imbedded in the turbulent FFS flow using proper orthogonal decomposition (POD).

Experiment

The particle image velocimetry experiments were performed in an Eiffel-type, open circuit, boundary layer wind tunnel with a freestream turbulence intensity of ~0.45%. The test section is 67×67 cm in cross-section and 3 m in length within which a 2.90 m long hydraulically smooth flat plate with an elliptically shaped leading edge is suspended 90 mm above the floor of the tunnel. The boundary layer is tripped by a cylindrical rod placed just downstream of the leading edge of the plate. The roughness topography of the rough-block's top surface is replicated from the profilometry measurements of a roughened turbine blade. High spatial resolution particle image velocimetry measurements are performed in the $x-y$ planes in turbulent boundary layers over both smooth and rough steps of the same mean heights at Re_h=3450 and δ/h=8. The resulting velocity vector fields have a grid spacing of 0.2 mm or h/32.

Snapshot POD

In POD analysis, a best estimation of a given instantaneous turbulent velocity field, $u(x,t)$, is determined in terms of N deterministic spatial POD modes $\phi_i(x)$, $i=1,\dots N$ and N random temporal functions $a_i(t)$, $i=1,\dots N$. When $N\rightarrow\infty$, the instantaneous velocity field can be fully reconstructed. For PIV measured velocity fields, solving for $\phi_i(x)$ directly requires significant amounts of computational efforts and therefore the more efficient snapshot POD is usually used to compute $a_i(t)$ first. In snapshot POD, the eigenvalue problem of the form

$$\lambda_i a_i(t) = \int_T \left(\int_\Omega u(x,t)u(x,t')dx \right) a_i(t')dt'$$

is solved for $a_i(t)$, where λ_i are eigenvalues, Ω is the spatial domain of interest, and T is the time domain which can be considered as the ensemble of velocity fields measured by PIV. POD modes $\phi_i(x)$ are subsequently determined by $\phi_i(x) = \frac{\int_T u a_i dt}{\int_T a_i^2 dt}$.

Preliminary Results

Snapshot POD was first applied to the fluctuating velocity fields downstream of the smooth and rough forward-facing steps with both streamwise (u) and wall-normal (v) velocity components. The fractional energy contributions of the first ten POD modes, $E_i \equiv \lambda_i/(\sum \lambda_i)$, to $\langle u^2 + v^2 \rangle$ are tabulated in table 1. It is seen that the first two modes in the rough FFS flow contain more energy than that in the smooth FFS flow.

POD Mode i	1	2	3	4	5	6	7	8	9	10
Smooth FFS	20.38%	6.68%	6.24%	3.79%	3.59%	2.95%	2.84%	2.63%	1.98%	1.71%
Rough FFS	27.37%	7.89%	4.56%	3.22%	3.04%	2.75%	2.02%	1.80%	1.64%	1.56%

Table 1: The contribution of the first ten POD modes to $\langle u^2 + v^2 \rangle$ for turbulent boundary layer flows over the smooth forward-facing step and the rough forward-facing step.

POD was then applied to streamwise and wall-normal velocity components individually to explore the effect of the highly irregular roughness topography on the flow structures of different scales that contribute significantly to Reynolds normal stresses $\langle u^2 \rangle$ and $\langle v^2 \rangle$, respectively. The fractional contributions of the first ten POD modes to $\langle u^2 \rangle$ and $\langle v^2 \rangle$ are given in tables 2 and 3, respectively. It is observed that the roughness causes more energy fractions in the first two POD modes indicating a more significant role played by large-scale structures in the rough FFS flow in producing Reynolds normal stresses.

POD Mode i	1	2	3	4	5	6	7	8	9	10
Smooth FFS	26.93%	8.60%	8.27%	4.26%	3.67%	3.47%	3.24%	2.63%	1.91%	1.77%
Rough FFS	34.55%	9.81%	5.17%	3.88%	3.53%	2.84%	2.31%	2.00%	1.82%	1.56%

Table 2: The contribution of the first ten POD modes to $\langle u^2 \rangle$ for turbulent boundary layer flows over the smooth forward-facing step and the rough forward-facing step.

POD Mode i	1	2	3	4	5	6	7	8	9	10
Smooth FFS	7.56%	5.39%	4.88%	4.18%	3.86%	3.60%	2.96%	2.83%	2.71%	2.40%
Rough FFS	11.95%	6.26%	4.82%	4.32%	3.75%	3.49%	2.76%	2.66%	2.31%	2.12%

Table 3: The contribution of the first ten POD modes to $\langle v^2 \rangle$ for turbulent boundary layer flows over the smooth forward-facing step and the rough forward-facing step.

Other results from the POD analysis such as eigenfunctions will also be studied to investigate the impacts of the present random roughness on the turbulence structures at different scales within the FFS flows.

Keywords

Turbulent boundary layer, forward-facing step flow, turbulence structures, proper orthogonal decomposition